AIAA JOURNAL
Vol. 30, No. 5, May 1992

Technical Notes

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Two- and Three-Dimensional Grid Generation by an Algebraic Homotopy Procedure

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Introduction

THE method to be described in this Note is an algebraic homotopy procedure that generates grids for Euler and Navier-Stokes flow computations for complex aerospace shapes. The present method is a modified version of a previous homotopic algorithm developed by the author.[1] The previous method had the capability for producing nearly orthogonal quasi-three-dimensional grid systems for use in finite volume Euler computations. Use of such grids in supersonic Euler computations has been recorded by the author in Refs. 2 and 3. A strict control over the spacing of grid lines near boundaries, however, was not obtainable in the previous method, thus limiting its use only to Euler computations. The present version incorporates improved blending techniques that result in control over both orthogonality and spacing at the body surface so that grids for Navier-Stokes computations can be generated. A description of the algorithm and representative grid examples are presented in the following sections.

Theory and Mathematical Development

Grid Generation

The procedure for grid generation consists of determining a family of curves representing a smooth and gradual transition from the given inner boundary (x_i, y_i) to the outer boundary (x_o, y_o) in two-dimensional planes at each z station. Assuming that the body surface coordinates are available, a distribution of a homotopy parameter η is specified. The η distribution may be specified by means of polynomials, exponents, trigonometric functions, etc., while ensuring that $\eta = 0$ on the body surface and $\eta = 1$ on the outer boundary. A shape transition function C for the grid is then specified by

$$C(\eta) = 1 - \eta^m \tag{1}$$

where m is a positive exponent that provides control over line spacing near boundaries. Thus, $C = 1$ on the inner boundary and $C = 0$ on the outer boundary with a smoothly varying distribution between boundaries. The function C is taken to be independent of z; it retains the same value at each z station. Essentially, the transition curves are defined by a family of maps given by the homotopy

$$\{h_\eta : A \rightarrow B \mid \eta \epsilon I\} \tag{2}$$

where I is the unit interval [0,1]. The inner and outer boundaries S_i and S_o are two homotopic maps such that $h_0 = S_i$ and $h_1 = S_o$. Coordinates of each boundary are expressed in terms of a parameter τ. For the inner boundary

$$\begin{aligned} x_i &= x_i(\tau) \\ y_i &= y_i(\tau) \end{aligned} \tag{3}$$

whereas identical expressions denote the coordinates of the outer boundary. In polar coordinates, τ could represent the angular coordinate. If y can be expressed as a single valued function of x, then $\tau = x$. For shapes of greater complexity, the choice of variables will vary. An arithmetic averaging between the boundaries yields the simplest family of grid lines given by

$$\begin{aligned} x(\eta,\tau) &= C(\eta)x_i(\tau) + [1 - C(\eta)]x_o(\tau) \\ y(\eta,\tau) &= C(\eta)y_i(\tau) + [1 - C(\eta)]y_o(\tau) \end{aligned} \tag{4}$$

Orthogonality and Spacing Control

The present method provides orthogonality and control over spacing while maintaining smoothness and preventing grid intersections by means of a technique for local perturbation of the homotopy parameter. The required amount of perturbation is derived from the boundary data exploiting the stability properties of homotopic maps under perturbation. A property is said to be stable provided that wherever $f_0 : x \rightarrow y$ possesses the property and $f_t : x \rightarrow y$ is a homotopy of f_0, then for some $\epsilon > 0$, each f_t with $t < \epsilon$ also possesses the property. The properties of grid smoothness and conformity of the overall grid with the given boundaries are stable under slight deformations of the map caused by small perturbations of the homotopy parameter. The basic interpolation scheme, Eqs. (4), may be written in a modified form as

$$\begin{aligned} x &= x_i E^p + x_o(1 - E^p) \\ y &= y_i E^q + y_o(1 - \epsilon^q) \end{aligned} \tag{5}$$

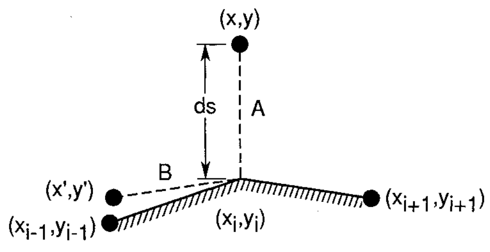

Fig. 1 Vectors used in orthogonalization.

Presented as Paper 90-1603 at AIAA 21st Fluid Dynamics, Plasma Dynamics, and Lasers Conference, Seattle, WA, June 18–20, 1990; received Aug. 5, 1991; revision received Aug. 14, 1991; accepted for publication Aug. 14, 1991.

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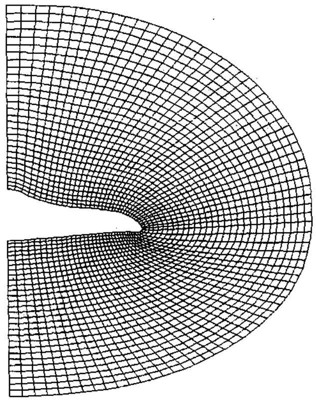

Fig. 2 Sectional grid with constant boundary spacing.

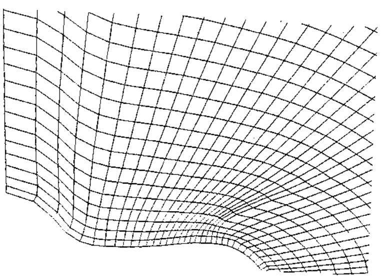

Fig. 3 Enlarged view of orthogonal sectional grid.

where

$$E = 1 - \eta^m \tag{6}$$

The subscripts i and o denote the inner and outer boundaries, respectively. Here, E plays a role analogous to that of a homotopy parameter. Modifications of E, therefore, cause slight deformations of a given map and may be used to achieve orthogonality and prescribed spacing at the inner boundary. This control is provided here through the use of the exponents p and q. These exponents are not constants with respect to τ and their values must be determined from the boundary data subject to the constraints of orthogonality and required spacing.

The orthogonality condition requires that the vectors A and B in Fig. 1 be orthogonal. The vector A is found by connecting the point (x_i,y_i) on the inner boundary and the point (x,y) lying just off the boundary on the trajectory in question. The second vector B passes through the point (x_i,y_i) and a point (x',y') on the line passing through (x_i,y_i) and parallel to the line joining (x_{i+1},y_{i+1}) and (x_{i-1},y_{i-1}) on the inner boundary. The orthogonality condition is that the dot product of the vectors A and B be zero, i.e.,

$$A \cdot B = 0 \tag{7}$$

which translates to

$$(x-x_i)(x'-x_i) + (y-y_i)(y'-y_i) = 0 \tag{8}$$

Substituting Eqs. (5) into Eq. (8), one obtains

$$(x_o-x_i)(1-E^p)(x'-x_i) + (y_o-y_i)(1-E^q)(y'-y_i) \tag{9}$$

The second condition, that of specified spacing ds in Fig. 1, can be written as

$$(x-x_i)^2 + (y-y_i)^2 = ds^2 \tag{10}$$

Substitution of Eqs. (5) into Eq. (10) results in

$$[(x_o-x_i)(1-E^p)]^2 + [(y_o-y_i)(1-E^q)]^2 = ds^2 \tag{11}$$

The exponents p and q can be solved for from Eqs. (9) and (11) and are found to be given by

$$p = \frac{\ln\{1 + [B(y'-y_i)/(x_o-x_i)(x'-x_i)]\}}{\ln E} \tag{12}$$

$$q = \frac{\ln\{1 - [B/(y_o-y_i)]\}}{\ln E} \tag{13}$$

where

$$B = \frac{ds}{\sqrt{1 + [(y'-y_i)^2/(x'-x_i)^2]}} \tag{14}$$

and E has the value corresponding to the homotopic curve lying next to the inner boundary.

Results and Discussions

A planar grid presented in Fig. 2 shows that the grid spacing adjacent to the inner boundary is constant along the circumference. The grid is also seen to be smooth, orthogonal at the inner boundary, and free from intersecting trajectories. It may be noticed in the grid presented in Fig. 3 that, although nearly orthogonal and nonintersecting grids can be generated by the present code, complex boundary shapes may cause local nonuniformities in the grid. In such complex cases, the algebraic grid produced by the present method may be postprocessed by an elliptic smoother.

References

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Efficiency of a Statistical Transport Model for Turbulent Particle Dispersion

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Introduction

A STATISTICAL transport model for turbulent particle dispersion was recently introduced in this journal[1] having significant potential for improving the computational efficiency and robustness of spray combustion CFD analyses. The technique was based on coupling a stochastic direct-modeling approach for parcel/eddy interaction properties with continuous probability density functions (pdf) to describe the physical particle temporal and spatial distribution. More specif-

Received March 21, 1991; revision received Aug. 6, 1991; accepted for publication Aug. 12, 1991.

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